Tip 1 – Absolute Value

- The absolute value of x, $|x|$, is regarded as the distance of x from zero.

- How do we convert the general interval into an expression using absolute value?

- Example: $10 \leq x \leq 30$

  - Step 1) Find the midpoint: $\frac{10+30}{2} = 20$

  - Step 2) Find the distance to either point: $20 - 10 = 10$

  - Step 3) Substitute: $|x - midpoint| \leq distance$

Tip 1 – Absolute Value (cont.)

- If $|x + 3| < 5$, what is the value of x?

- If $|x + 3| > 5$, what is the value of x?

Tip 1 – Absolute Value (cont.)

- At a bottling company, a computerized machine accepts a bottle only if the number of fluid ounces is greater than or equal to $5\frac{3}{7}$ and less than or equal to $6\frac{4}{7}$. If the machine accepts a bottle containing f fluid ounces, which of the following describes all possible values of f?

    A. $|f - 6| < \frac{4}{7}$

    B. $|f - 6| < \frac{3}{7}$

    C. $|f + 6| > \frac{4}{7}$

    D. $|6 - f| \le \frac{4}{7}$

    E. $|f + 6| \le \frac{4}{7}$

Tip 1 – Absolute Value (cont.)

- At a milk company, Machine X fills a box with milk, and machine Y eliminates the milk-box if the weight is less than 450 grams, or greater than 500 grams. If the weight of the box that will be eliminated by machine Y is E, in grams, which of the following describes all possible values of E?

  A. $|E - 475| < 25$
  B. $|E + 475| < 25$
  C. $|E - 500| > 450$
  D. $|475 - E| = 25$
  E. $|E - 475| > 25$

Tip #2 - Ratio to Similar Figures

- Two polygons are similar if and only if their corresponding angles are congruent and their corresponding sides are in proportion

- If the ratio of the corresponding lengths is a:b, then the ratio of the areas is $a^2 : b^2$ and the ratio of the volumes is $a^3 : b^3$.

Ratio to Similar Figures (cont)

- The ratio of the sides of 2 similar triangles is 5:2. If the area of the larger triangle is 30, what is the area of the smaller triangle?

- Solution
  - The ratio of areas is 25:4
  - 25k = 30 or k = 1.2
  - Therefore 4k = 4(1.2) = 4.8

Ratio to Similar Figures (cont.)

- In Triangle ABC, AB, PQ, & RS are parallel and the ratio of the lengths is AQ:QS:SC = 2:2:3. If the area of quadrilateral PRSQ is 48, what is the area of Triangle ABC?

  A. 84
  B. 92
  C. 105
  D. 144
  E. 147
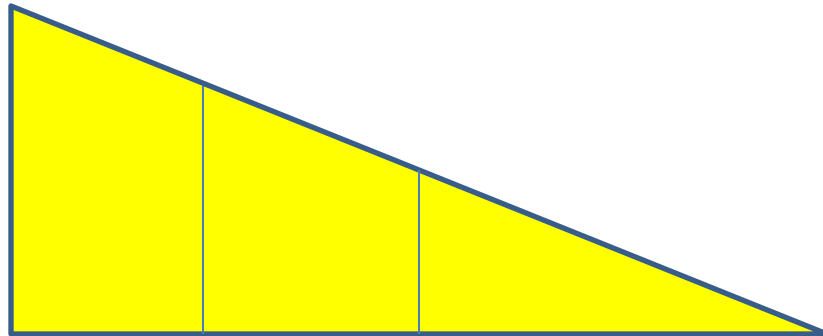
Tip 3: Combined Range of Two Intervals

Rules

If $5 \leq A \leq 10 \; and \; 2 \leq B \leq 5,$

    i. $7 \leq A + B \leq 15$

    ii. $10 \leq A \times B \leq 50$

    iii. $0 \leq A - B \leq 8$

    iv. $1 \leq \frac{A}{B} \leq 5$

Smallest value $\leq$ Combined Range $\leq$ Largest Value

Example 1

- Given $2 \leq P \leq 8 \; and \; 1 \leq Q \leq 4$. By how much is the maximum of $\frac{P}{Q} >$ the minimum of $\frac{P}{Q}$?

    i.   Solution: max is $\frac{8}{1} = 8$

    ii.   Min is $\frac{2}{4} = \frac{1}{2} = .5$

    iii. $8 - .5 = 7.5$

Tip 3: Combined Range of Two Intervals (cont.)

- If $-2 < x < 4 \; and \; -3 < y < 2$, what are all possible values of $x - y$?

  A. $-4 < x - y < 2$

  B. $1 < x - y < 7$

  C. $1 < x - y < 4$

  D. $-4 < x - y < 7$

  E. $-5 < x - y < 7$

Tip 3: Combined Range of Two Intervals (cont.)

- The value of p is between 1 and 4, and the value of q is between 2 and 6. Which of the following is a possible value of $\frac{q}{p}$?

  A. Between $\frac{1}{2}$ and $\frac{2}{3}$

  B. Between $\frac{2}{3}$ and 2

  C. Between $\frac{1}{2}$ and 6

  D. Between 2 and $\frac{1}{2}$
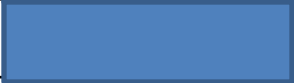
  E. Between $\frac{1}{2}$ and $1\frac{1}{2}$

Tip 4: Classifying a Group in Two Different Ways

- ***Organize the information in a table and use a convenient number***
  - Example: In a certain reading group organized of only senior and junior students, 3/5 of the students are boys, and the ratio of seniors to juniors is 4:5. If 2/3 of girls are seniors, what fraction of the boys are juniors?

	BOYS	GIRLS	
Seniors			4/9
Juniors			5/9
	3/5	2/5	1

Tip 4: Classifying a Group in Two Different Ways (cont.)

- On a certain college faculty, 4/7 of the professors are male, and the ratio of the professors older than 50 years to the professors less than or equal to 50 years is 2:5. If 1/5 of the male professors are older than 50 years, what fraction of female professors are less than or equal to 50 years.

	Males	Females	
> 50			2
≤ 50			5
	4	3	7

Tip 5: Direct Variation

- When 2 variables are related in such a way that $y = kx$, the 2 variables are said to be in direct variation.

- Expression of direct variation:
  - $i.$ $y = kx$
  - $ii.$ $\frac{y}{x} = k$

- Geometric interpretation: $y = kx$ is a special linear equation where the $y - intercept$ is (0, 0).

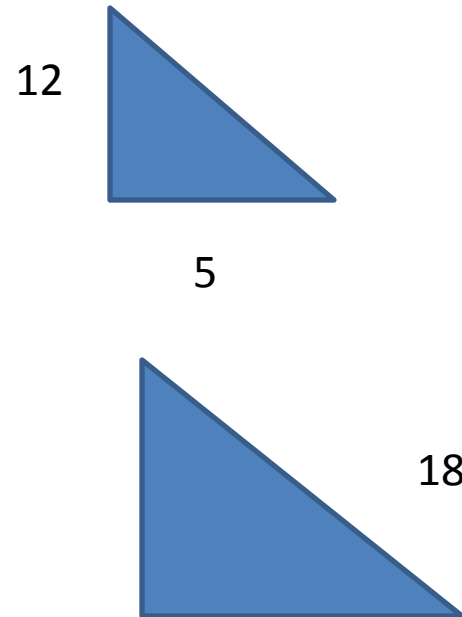- In the $xy - $ coordinate plane, $y = kx$, where $k\ is\ slope,$ but $y - intercept$ must be zero.

Tip 5: Direct Variation (cont.)

- The value y changes directly proportional to the value of x. If y = 15 when x = 5, what is the value of y when x = 12.5?

Tip 5: Direct Variation (cont.)

- The 2 adjoining triangles are similar. What is the length of side DF?
  - 6
  - $\dfrac{82}{13}$
  - $\dfrac{90}{13}$
  - 8
  - $\dfrac{100}{13}$

12

5

18

Tip 6: Inverse Variation

- When 2 variables are related in such a way that $xy = k$, the two variables are said to be in inverse variation.
  - Properties:
    - The value of two variables change in an opposite way; that is, as one variable increases, the other decreases.
   - The product k is unchanged

Tip 7: Special Triangles

- Angle-based Right Triangle
  - 30-60-90 triangle
    - In a triangle whose angles are in the ratio 1:2:3, the sides are in the ratio $1, \sqrt{3}, 2$
  - 45-45-90 Triangle
    - In a triangle whose 3 angles are in the ratio 1:1:2, the sides are in the ratio $1, 1, \sqrt{2}$
- Side-based triangles
  - Right triangles whose sides are Pythagorean triples as follows.

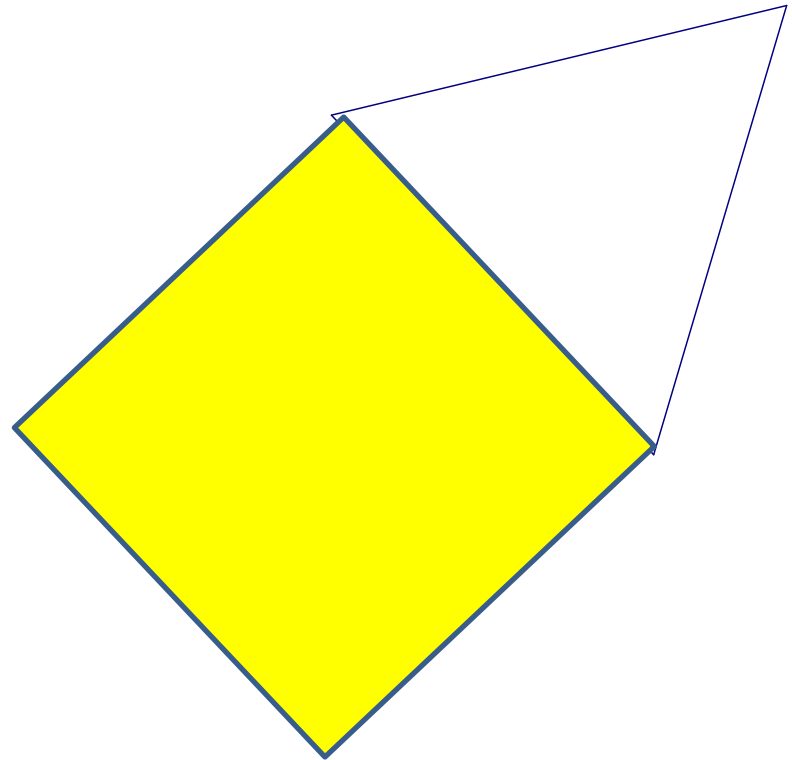| 3:4:5	5:12:13	8:15:17
7:24:25	9:40:41	11:60:61

Tip 7: Special Triangles (cont.)

- An equilateral triangle ABC is inscribed inside a circle with radius = 10.
  - What is the area of ΔABC?
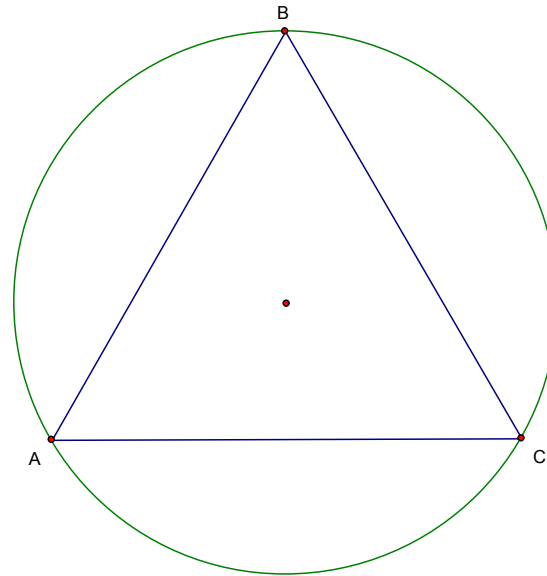
Tip 7: Special Triangles (cont.)

- Figure ABDE is a square and ΔBCD is an <u>equilateral</u> triangle. If the area of ΔBCD is $16\sqrt{3}$

  - What is the area of the square?
    - A. 32
    - *B.* $32\sqrt{3}$
    - C. 64
    - *D.* $64\sqrt{2}$
    - E. 72

Tip 8: Exponents

- The exponent is the number of times the base is used as a factor.

$$5^2 = 25 \begin{cases} 5 = base \\ 2 = exponent \\ 25 = power \end{cases}$$

- The mathematical operations of exponents are as follows:

1. $a^m \cdot a^n = a^{m+n}$
2. $(a^m)^n = a^{m \cdot n}$
3. $(ab)^m = a^m \cdot b^m$
4. $a^{-m} = \dfrac{1}{a^m}$
5. $a^0 = 1$
6. $\dfrac{a^m}{a^n} = a^{m-n}$
7. $\left(\dfrac{a}{b}\right)^m = \dfrac{a^m}{b^m}$
8. $\sqrt[n]{a^m} = a^{\frac{m}{n}}$

Tip 8: Practice

1. If $\{(-2)^3 \cdot 8^2\}^4 = (2^4)^n$, what is the positive value of **n**?

   A. **6**

   B. **7**

   C. **8**

   D. **9**

   E. **10**

Tip 8: Practice

- If $4^3 + 4^3 + 4^3 + 4^3 = 2^n$, what is the value of *n*?

    A. 2

    B. 4

    C. 6

    D. 8

    E. 10

Tip 8: Practice

- If m and n are positive and $5m^5n^{-3} = 20m^3n$ what is the value of m in terms of n?

$A. \quad \dfrac{1}{4n}$

$B. \quad \dfrac{4}{n^2}$

$C. \quad \dfrac{4}{n^3}$

$D. \quad 2n^2$

$E. \quad 4n^2$

Tip 8: Practice

- If a and b are positive integers, $(a^{-4}b)^{-1} = 16, and\ b = a^2$, which could be true of the value of a?

  a. 0

  b. 2

  c. 4

  d. 8

  e. 12

Tip 8: Practice

- If $k^{-2} \times 2^3 = 2^7$, *what is the value of k?*
  A. 2
  B. 4
  C. 8
  D. $\frac{1}{4}$
  E. $\frac{1}{8}$

Tip 8: Practice (cont.)

- If p and q are positive integers, $p^{-3} = 2^{-6}$, and $q^{-2} = 4^2$, what is the value of pq ?

  a. 1
  b. 2
  c. 3
  d. 4
  e. 5

Tip 9: Geometric Probability

- Geometric Probability is the probability dealing with the areas of regions instead of the "number" of outcomes. The equation becomes

  - $Probability = \dfrac{Favorable\ Region}{Area\ of\ Total\ Region}$

Tip 8: Practice (last 1)

- If a and b are positive integers and $(a^6 b^4)^{\frac{1}{2}} = 675$, what is the value of $a + b$?

  a. 3

  b. 4

  c. 5

  d. 7

  e. 8